Question 12 (7 marks)

<EFOFEX>
id:fxd{edf54efd-7f3f-4cd5-b6a4-f139dd2de51c}

FXData:
</EFOFEX>The slope field for the differential equation

where is a constant, is shown at right.

(a) Use a feature of the slope field to explain why and hence determine the slope at the  
point . (2 marks)

(b) Determine the solution of the differential equation that contains the point in the form . (4 marks)

(c) Sketch the solution curve that contains the point on the slope field. (1 mark)

**Question 12 (7 marks)**

<EFOFEX>
id:fxd{5441104e-5779-4279-95ec-c859773c7073}

FXData:
</EFOFEX>The slope field for the differential equation

where is a constant, is shown at right.

|  |
| --- |
| Solution (c) |
| See graph |
| Specific behaviours |
| ✓ ‘normal’ curve thru’ |

(a) Use a feature of the slope field to explain why and hence determine the slope at the  
point . (2 marks)

|  |
| --- |
| Solution |
| When and it can be seen that and so .  Slope at is . |
| Specific behaviours |
| ✓ explains using at  ü correct slope at |

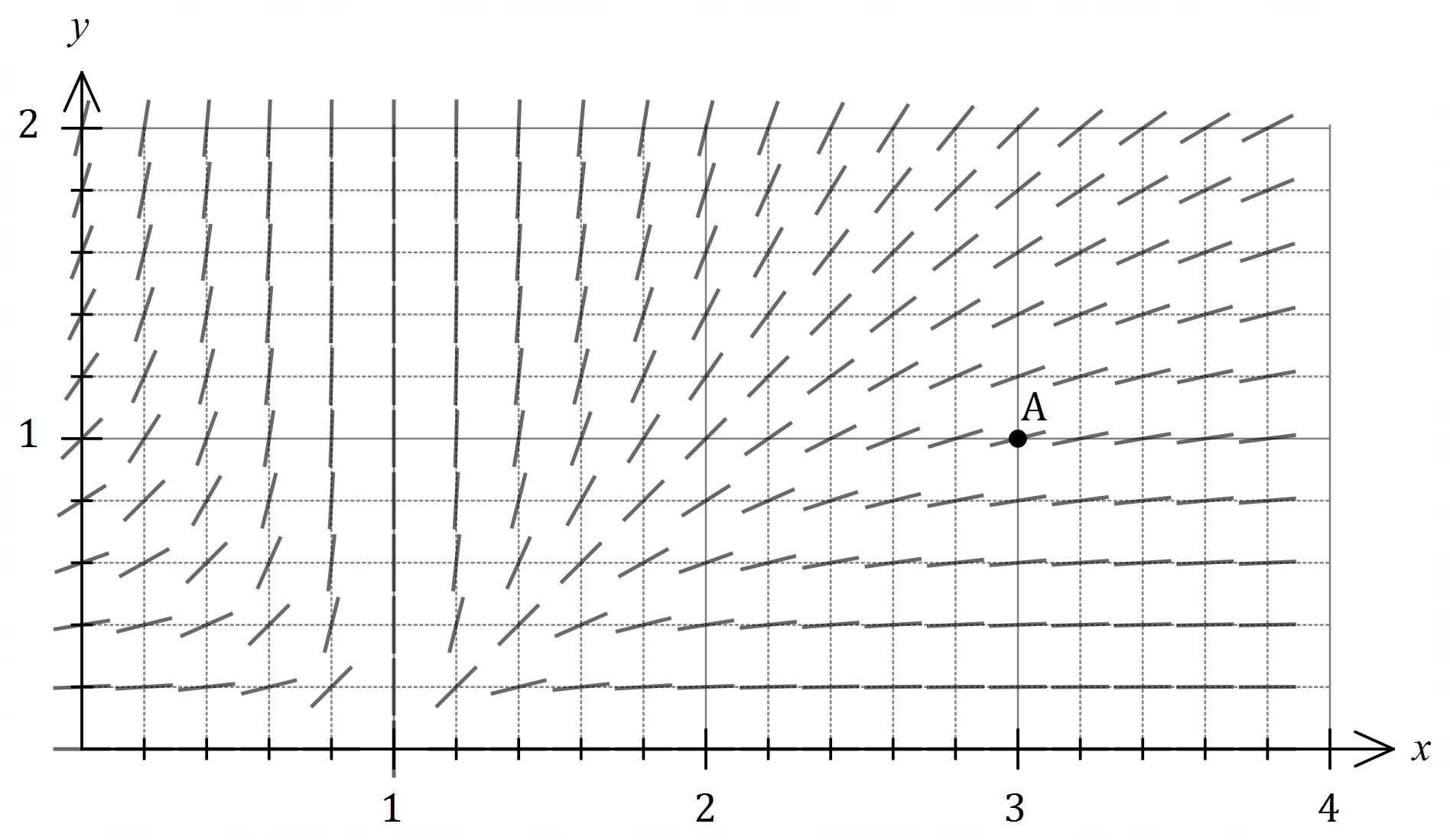
(b) Determine the solution of the differential equation that contains the point in the form . (4 marks)

|  |
| --- |
| Solution |
| At , and so require |
| Specific behaviours |
| ✓ separates variables and antidifferentiates  ü recognises that to replace  ü evaluates constant  ü correctly expresses as a function of |

(c) Sketch the solution curve that contains the point on the slope field. (1 mark)

Question 16 (6 marks)

The graph below shows the slope field for the differential equation .



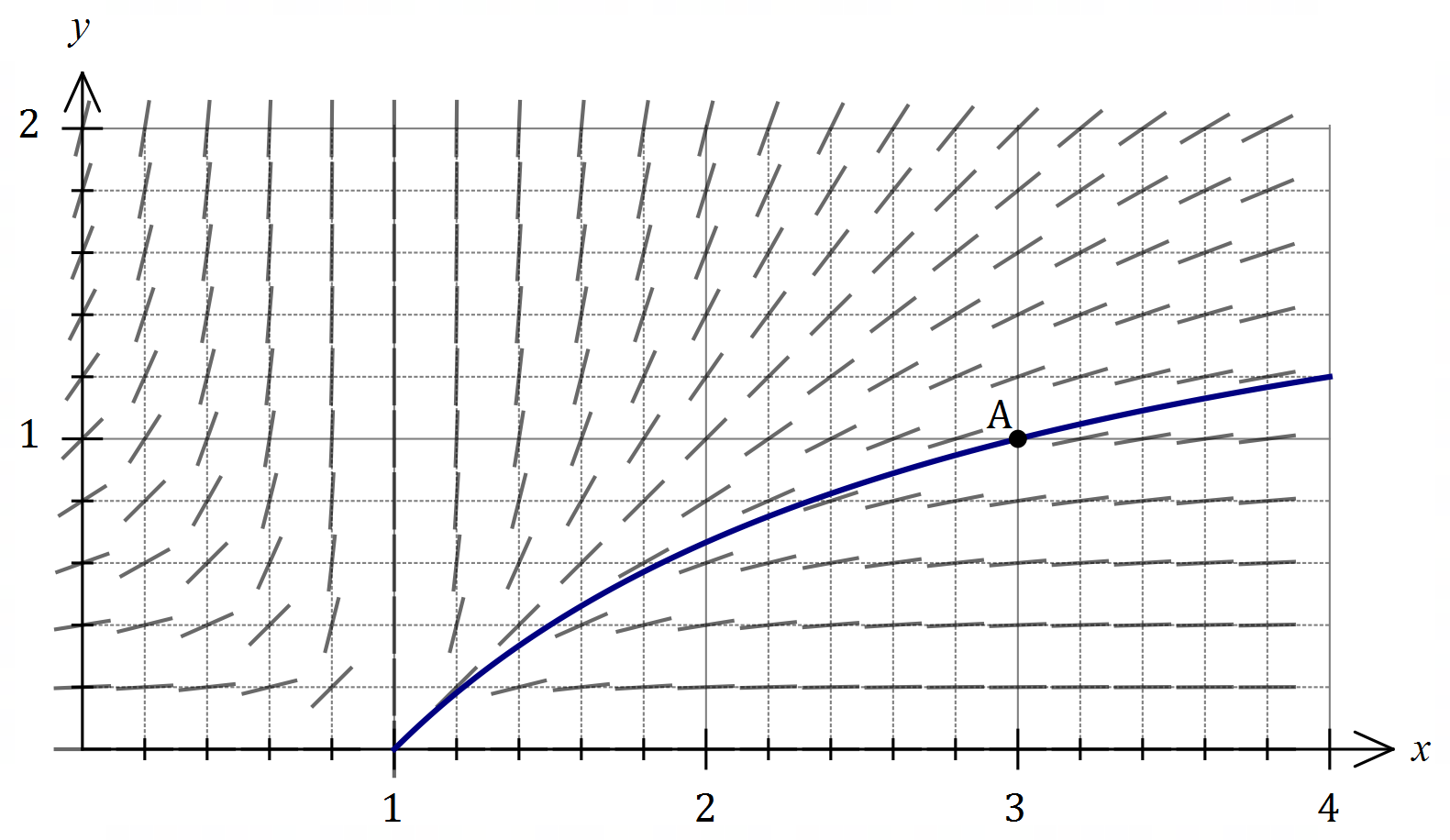
(a) Determine the value of the slope field at point . (1 mark)

(b) On the axes above, sketch the solution curve for the differential equation that passes through point . (1 mark)

(c) Determine the particular solution to the differential equation that has initial solution . (4 marks)

Question 16 (6 marks)

The graph below shows the slope field for the differential equation .



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ curve through , follows slopes, to RH edge |

(a) Determine the value of the slope field at point . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

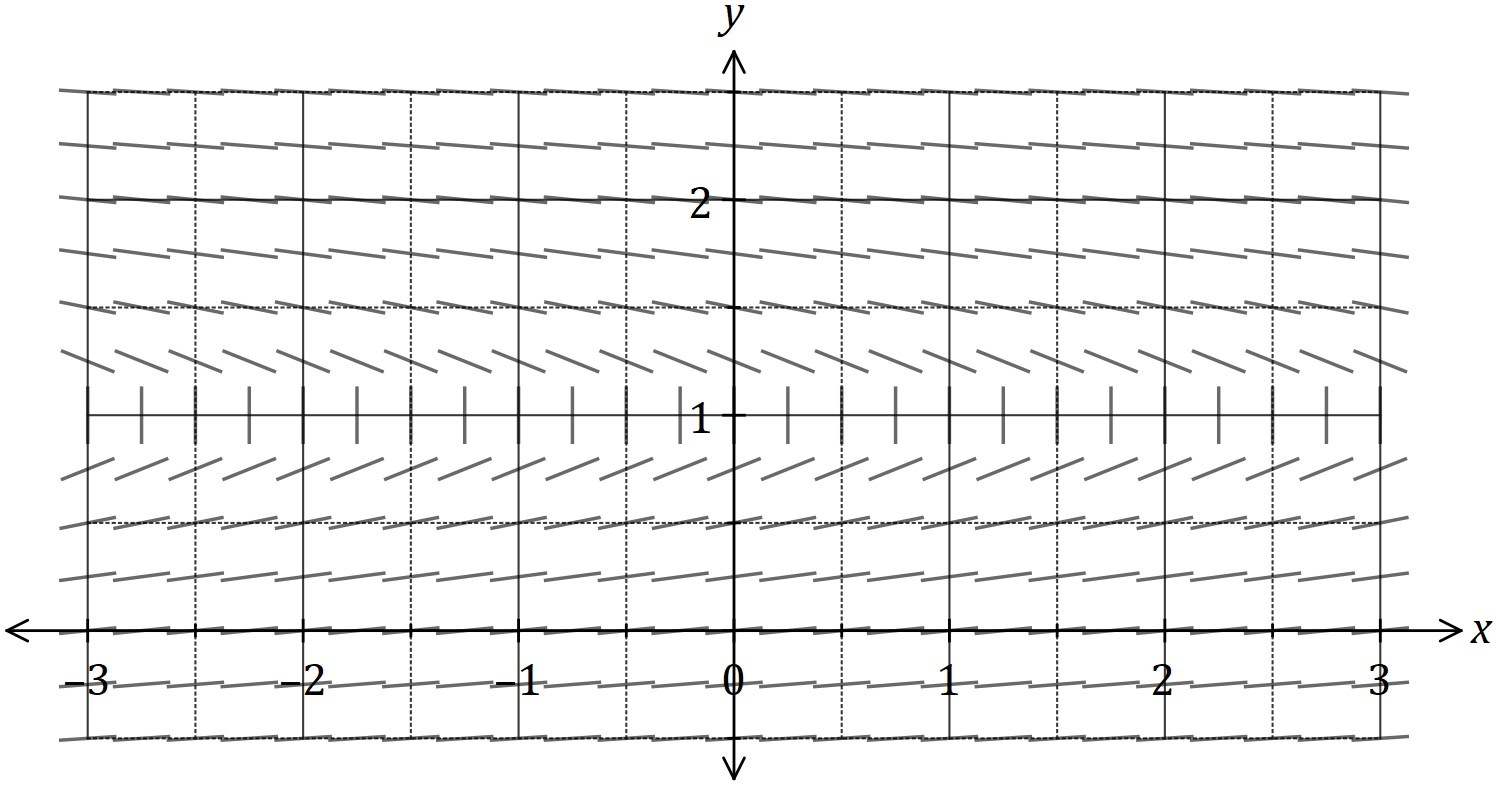
(b) On the axes above, sketch the solution curve for the differential equation that passes through point . (1 mark)

(c) Determine the particular solution to the differential equation that has initial solution . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ separates variables   antidifferentiates both sides with constant   evaluates constant   expresses in form |

Question 14 (8 marks)

The slope field for the differential equation is shown below.



(a) Sketch the solution of the differential equation that passes through the point .

(2 marks)

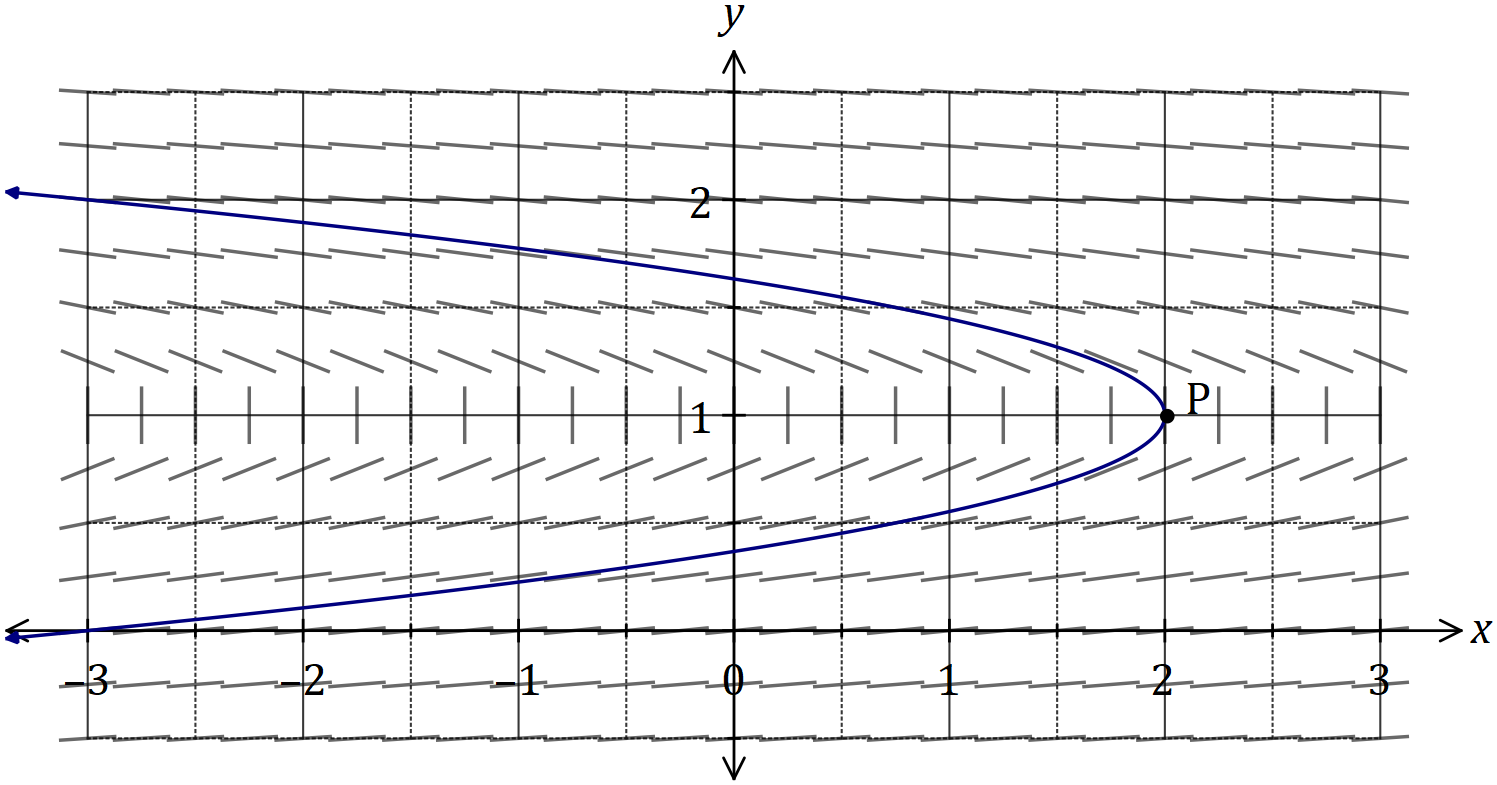
A different solution of the differential equation passes through the points and .

(b) Use the increments formula to estimate the value of . (3 marks)

(c) Calculate the value of the second derivative of the solution through and use it to explain whether your solution to (b) is an under or over estimate. (3 marks)

Question 14 (8 marks)

The slope field for the differential equation is shown below.



|  |
| --- |
| **Solution (a)** |
| See graph |
| **Specific behaviours** |
| ✓ vertex at   parabola, symmetrical about |

(a) Sketch the solution of the differential equation that passes through the point .

(2 marks)

A different solution of the differential equation passes through the points and .

(b) Use the increments formula to estimate the value of . (3 marks)

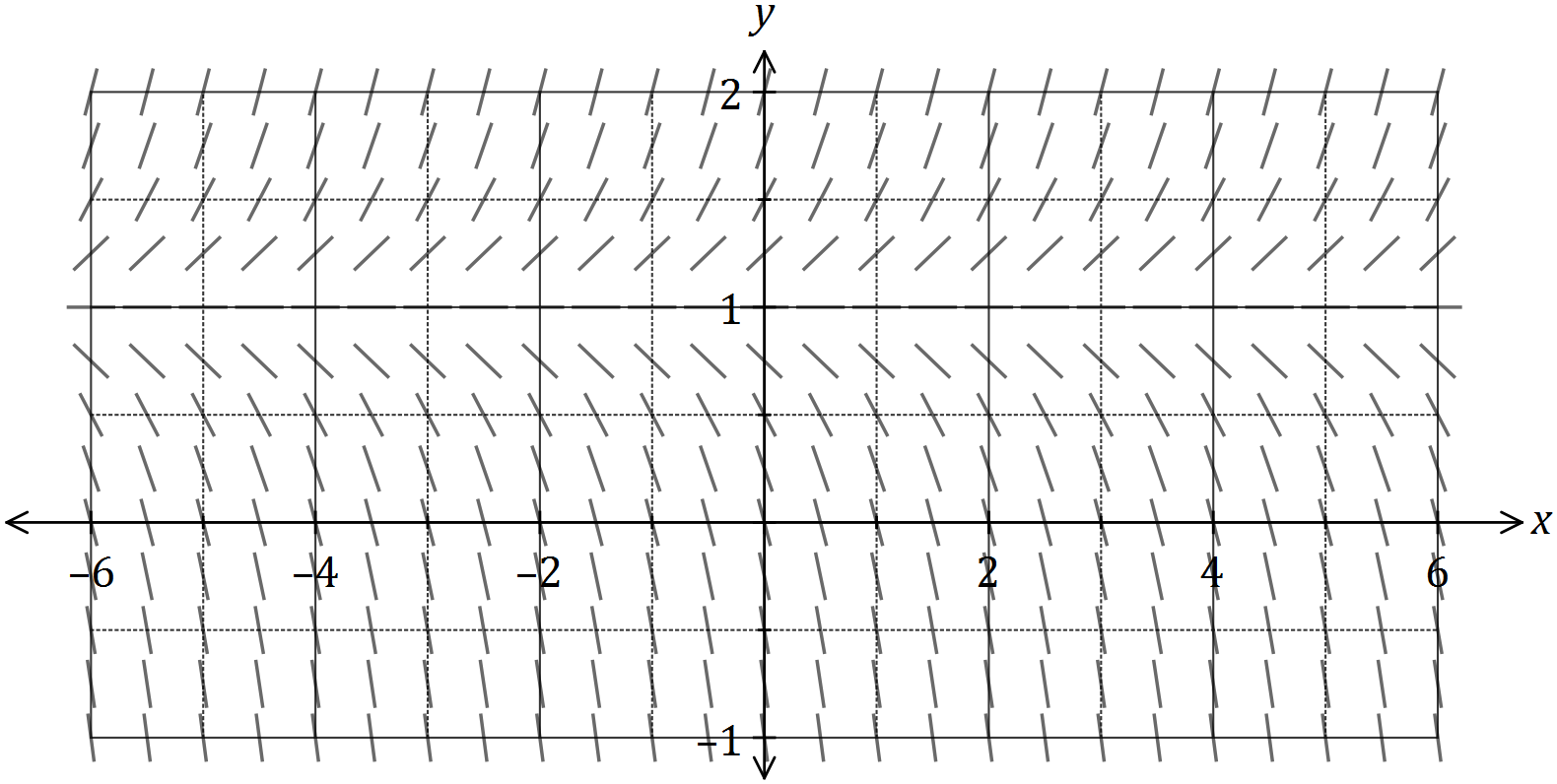
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates gradient at   calculates using increments formula   correct estimate |

(c) Calculate the value of the second derivative of the solution through and use it to explain whether your solution to (b) is an under or over estimate. (3 marks)

|  |
| --- |
| **Solution** |
| Curve is concave up, so will be an under estimate. |
| **Specific behaviours** |
| ✓ expression for second derivative   correct value   correct deduction |

Question 20 (9 marks)

A first-order differential equation has a slope field as shown below.



(a) Sketch the solution of the equation that passes through , where the value of the slope is -2. (3 marks)

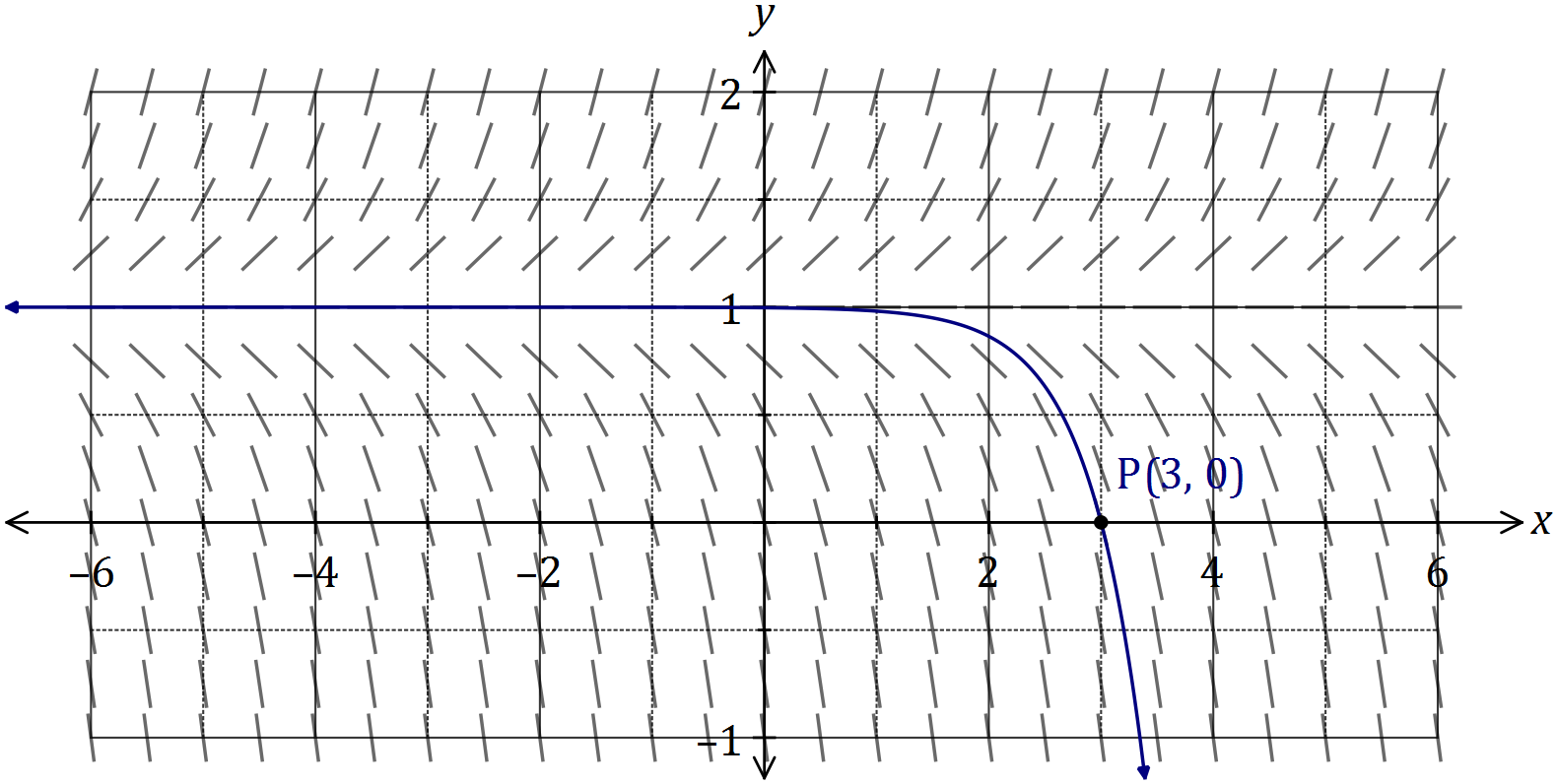
(b) The general differential equation for the slope field is of the form below, where and are constants:

Determine the solution to this equation that passes through in the form .

(6 marks)

Question 20 (9 marks)

A first-order differential equation has a slope field as shown below.



|  |
| --- |
| **Solution (a)** |
| See graph |
| **Specific behaviours** |
| ✓ through (3, 0), close to (3.5, -1)  ✓ y-intercept close to (0, 1)  ✓ asymptotic with |

(a) Sketch the solution of the equation that passes through , where the value of the slope is -2. (3 marks)

(b) The general differential equation for the slope field is of the form below, where and are constants:

Derive the solution to this equation that passes through in the form . (6 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates value of  ✓ determines value of  ✓ separates variables and antidifferentiates  ✓ determines constant  ✓ considers and adjusts  ✓ expresses as required |

|  |
| --- |
| **Solution** |
| Using CAS dsolve(), max 3/6 |
| **Specific behaviours** |
| ✓ indicates value of  ✓ determines value of  ✓ expresses as required |